59. v = velocity

$v = c(r_0 - r)r^2$ $\quad \frac{r_0}{2} \le r \le r_0$

$v = c(r_0 r^2 - r^3)$ $\quad r_0$ = rest radius

a) v max when $r = \frac{2}{3}r_0$ $\quad c$ = constant (positive)

$v' = c(2r_0 r - 3r^2) = 0$

$r(2r_0 - 3r) = 0$

$r \neq 0$ $\quad 2r_0 = 3r$

$r = \frac{2}{3}r_0$

$v'' = c(2r_0 - 6r) \Big| r = \frac{2}{3}r_0$

$c(2r_0 - 6\cdot\frac{2}{3}r_0) = c(2r_0 - 4r_0) < 0$

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41. $y = \sqrt{x}$ $\quad (\frac{3}{2}, 0)$

$(x, \sqrt{x})$, d, $(\frac{3}{2}, 0)$

$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$d^2 = (\sqrt{x} - 0)^2 + (x - \frac{3}{2})^2$

$d^2 = x + (x - \frac{3}{2})^2$

$d^2 = 1 + (1 - \frac{3}{2})^2$

$d^2 = 1 + \frac{1}{4}$

$d = \sqrt{\frac{5}{4}}$

$(d^2)' = 1 + 2(x - \frac{3}{2})$

$0 = (d^2)' = 1 + 2x - 3 = 2x - 2$

$x = 1$ $\quad (d^2)'' = 2$ (++ ☺)

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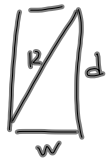
37. s = strength

d = depth

w = width



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4.4c Modeling and Optimization

Examples from Economics

Maximum Profit: If there is a maximum profit, it occurs when marginal revenue = marginal cost

$P = R - C$

$P' = R' - C' = 0$

$R' = C'$

Oct 25-8:00 AM

Suppose r(x) = 9x and c(x) = x^3 - 6x^2 + 15x, where x represents 1000's of units. Is there a production level that maximizes profit? If so, what is it?

$P = 9x - (x^3 - 6x^2 + 15x)$

$P' = 9 - (3x^2 - 12x + 15) = 0$

$x = 0.5858, 3.414$

$- \quad 0 \quad + \quad 0 \quad -$

$.5858 \qquad 3.414$

$P'' = -6x + 12$ $\quad x = 3.414 < 0$

max at 3.414 ☹

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average cost = c(x)/x

Minimum Average Cost: If there is a minimum average cost, it occurs when average cost = marginal cost.

$A = \frac{c(x)}{x}$

$A' = \frac{x \cdot c'(x) - c(x)}{x^2} = 0$

$x c'(x) - c(x) = 0$

$x c'(x) = c(x)$

$c'(x) = \frac{c(x)}{x}$

Oct 25-8:06 AM

Suppose c(x) = x^3 - 6x^2 +15x, where x represents 1000's of units. Is there a production level that minimizes average cost? If so, what is it?

$C'(x) = \frac{C(x)}{x}$

$3x^2 - 12x + 15 = \frac{x^3 - 6x^2 + 15x}{x}$

$3x^2 - 12x + 15 = x^2 - 6x + 15$ ave cost

$2x^2 - 6x = 0$

$2x(x-3) = 0$

$x = 0$ or $x = 3$

$A' = 2x - 6$

$A'' = 2$ (+ +)

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